CHAPTER 1

HOW SHOULD VOTES BE CAST AND COUNTED?

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1.1 Introduction

Particularly in the United States, where there is little use of other systems, there is a tendency to think of "voting," when only one candidate is to be elected, as a matter of inviting every voter to specify which candidate will receive his or her vote, counting the votes, and declaring the candidate with the most votes the winner. But voting is more complicated. Instead of inviting each voter to specify the candidate who will receive his or her vote, each voter can be asked for a ranking of the candidates. There are myriad ways that such ballots can be counted. Alternatively, each voter can be asked to approve or not approve each candidate, with the candidate receiving the most approvals winning. As yet another possibility, each voter can be asked to give a score to each candidate, with several possibilities as to how the votes are then processed. This chapter examines how a collectivity might decide what voting procedure to use.

Some important aspects of selecting a voting procedure are not discussed here. These include: How will candidates be selected? Who should be permitted to vote? Will voting be open or secret? If voting is secret, how will the integrity of the vote-counting process be ensured? If there is a tie, how will the tie be resolved? If there is a dispute, how will it be adjudicated? All these questions are important and need to be addressed in any voting environment, but they are not addressed here. This chapter is concerned with the choice of a form of voting and a rule for counting votes.

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1.2 Criteria for Evaluating Voting Rules

There is a tradition of evaluating voting procedures in terms of their logical properties: If there is a candidate who has a majority over every other candidate when compared head to head, does that candidate always win? Can it happen that by voting for a candidate, a voter can cause that candidate to lose? These and many other logical properties of voting rules have been studied.[1] The difficulty with evaluating voting procedures in terms of these logical properties is that such evaluations do not lead to conclusions. No voting procedure has all the properties that are attractive, so trade-offs must be made. And while it is possible to make trade-offs in terms of logical properties, such a procedure takes no account of the frequency with which troubling examples of different types arise under different rules. Therefore, it is sensible to evaluate voting rules by a smaller set of primarily nonlogical properties. The following seven characteristics of voting rules are used to evaluate voting rules in this chapter:

1. Sincere efficiency—the capacity to select the candidate that would yield the greatest total utility if voters voted sincerely.
2. Resistance to strategy—the capacity to avoid giving voters opportunities whereby a group of them, by voting insincerely, can obtain an outcome they prefer.
3. Resistance to tying.
4. Resistance to clones.
5. Ease of casting votes.
6. Ease of counting votes.
7. Ease of understanding how votes are counted.

The first three properties are measured by examining the performance of voting rules on masses of data derived from opinion surveys or mathematical models. The measures reported here of sincere efficiency and resistance to strategy come from Green-Armytage et al. (2016) and are based on German Politbarometer surveys between 1979 and 2008. Politbarometer surveys German voters about once a month, with an average of about 1,000 voters contacted for each survey. Among the many questions that are asked are ones that request evaluations of an average of about a dozen politicians on a scale of −5 to 5. The rankings provided by these scores are the input for evaluating voting rules. The inquiries reported here were undertaken by examining the 126,637 "simulated elections" that can be formed by comparing all combinations of three candidates from the same survey. Sincere efficiency is then measured by the proportion of instances in which the candidate selected by a voting rule is the one with the highest average score. Resistance to strategy is measured by the proportion of instances in which there is no subset of voters who would be able to secure the election of a candidate they preferred by coordinated strategic voting.

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The third property, resistance to tying, is measured by the proportion of instances in which a voting rule reaches a result without calling upon a tie-breaking procedure. Most of the measures of resistance to tying reported here are 1 minus the frequency of ties reported by Plassmann and Tideman (2014). These scores are based on a mathematical model that was calibrated to Politbarometer data and in particular on simulated elections with 100 voters. For the voting rules considered here that were not considered by Plassmann and Tideman (2014), estimates of resistance to tying are reported in italics and are based primarily on similarity to other voting rules.

The fourth property, resistance to clones, is the one formal property considered in this chapter. A voting rule has full resistance to clones (a score of 5) if the introduction of a clone (a candidate who is ranked by all voters immediately adjacent to another candidate) cannot affect the outcome except by replacing one clone with another. A voting rule has partial resistance to clones (a score of 3) if it can be affected only by clones that come in groups of three or more. If a voting rule can be affected by a single pair of clones, it receives a score of 1. When a voting rule lacks resistance to clones, the introduction of a clone can either improve or worsen prospects for the clones. Resistance to clones for most of the voting rules discussed here was reported in Tideman (2006).

The last three properties are impressionistic. How easy is it to vote? How easy is it to count the votes? How easy is it for voters to understand the vote-counting process? The rationales for these scores will be explained in the text. Since the last property, ease of understanding, is one that an expert could easily misjudge, the reported scores are the consensus of two representatives of the Oregon League of Women Voters (Peggy Bengry and James Ofsink) who were studying voting reform and offered their views in a conversation with the author.

1.3 Possible Rules and Their Scores

A very large number of voting rules have been proposed. Instead of trying to evaluate all or nearly all of them, this chapter makes a first cut by taking the rules near the frontier of possibilities with respect to sincere efficiency and resistance to strategy, as reported in Green-Armytage et al. (2016). In addition, we've included Plurality because of its familiarity and popularity. Table 1.1 gives the scores for 18 voting rules with respect to the seven properties just mentioned. Thirteen of the rules are listed in alphabetical order. Five appear as indented in the table following Minimax because they yield the same results as Minimax for elections with two or three candidates. For election with more than three candidates, they can yield results different from Minimax only if, at the top of the collective ranking, there is a "majority rule cycle." (A majority rule cycle is a head-to-head comparison in which every candidate loses to at least one of the other candidates in the cycle.) The information in table 1.1 regarding sincere efficiency and resistance to strategy is shown graphically in figure 1.1. Figure 1.2 shows, in more detail, the information from the upper-right corner of figure 1.1.

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Table 1.1 Seven Properties of 18 Voting Rules

Rule	Sincere Efficiency	Resistance to Strategy	Resistance to tying	Resistance to clones	Ease of Voting	Ease of Counting	Ease of Understanding
Approval	0.9537	0.7188	*0.9900*	5	5	4	4.5
Black	0.9524	0.7692	0.9977	1	3	3	2.5
Borda	0.9542	0.6226	0.9907	1	3	3.5	4
Condorcet-Anti-Plurality	0.9524	0.8606	*0.9750*	1	3	3	4
Condorcet-Hare	0.9516	0.9804	0.9746	5	3	2	2
Hare	0.9445	0.9804	0.9748	5	3	2.5	2.5
Minimax	0.9519	0.8461	0.9749	3	3	2.5	2
Kemeny	0.9519	0.8461	0.9749	1	3	0.5	1.5
Ranked Pairs	0.9519	0.8461	0.9749	5	3	1.5	1.5
Schulze (Beatpath)	0.9519	0.8461	0.9749	5	3	1.5	1
Simplified Dodgson	0.9519	0.8461	0.9749	1	3	2.5	2
Young	0.9519	0.8461	0.9749	3	3	0.5	1
Nanson	0.9519	0.8464	0.9766	1	3	2.5	2
Normalized. Cardinal Pairwise	0.9517	0.9042	*0.9980*	5	3	1	0.5
Normalized Range	0.9550	0.7029	*0.9980*	5	3	3.5	3
Plurality	0.9027	0.7749	0.9856	1	5	4	4.5
Random Dictator	0.5181	1.0000	—	5	5	5	5
Range	1.0000	0.1814	*0.9980*	5	3	3.5	4

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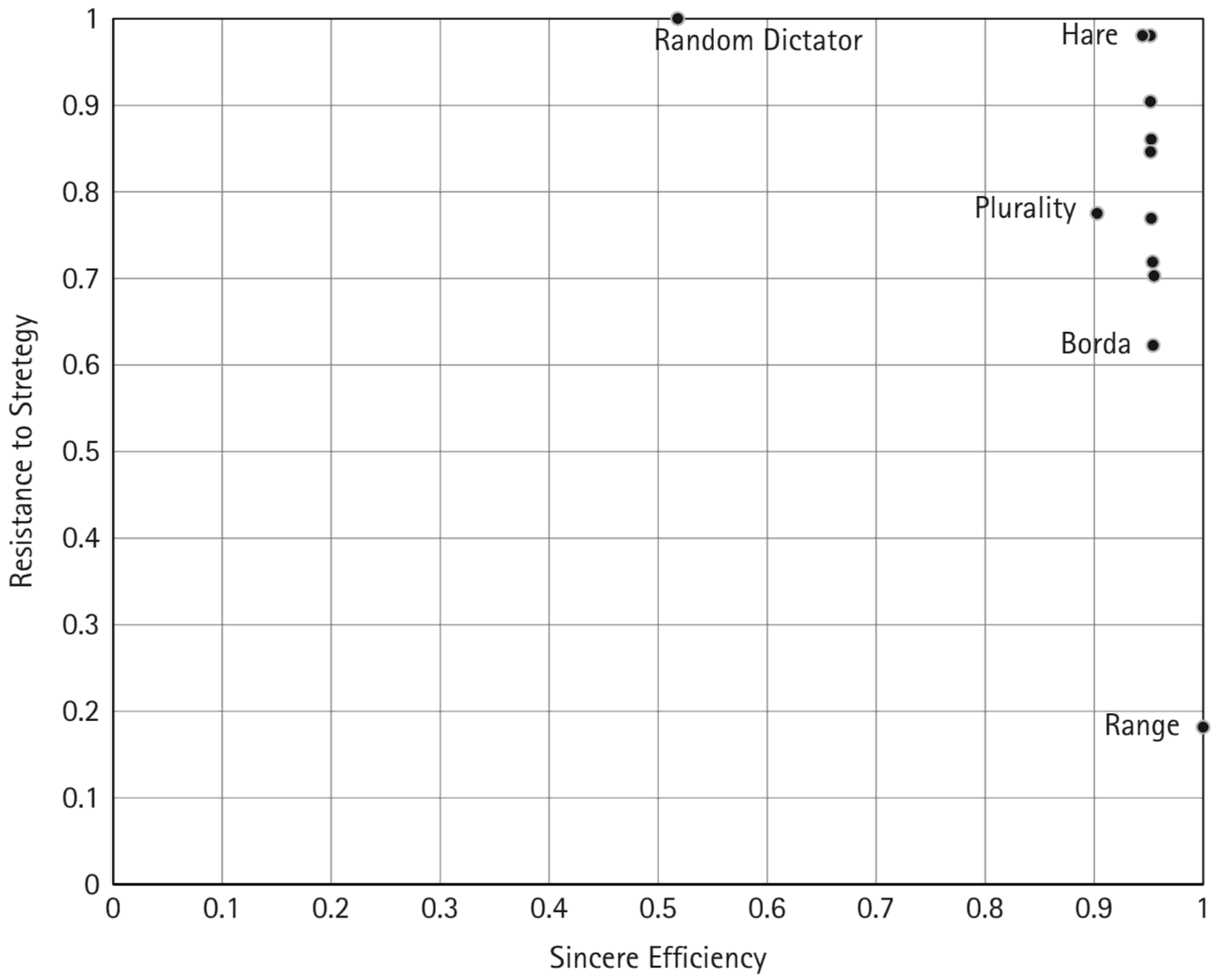

FIGURE 1.1 Sincere Efficiency and Resistance to Strategy for 18 Voting Rules

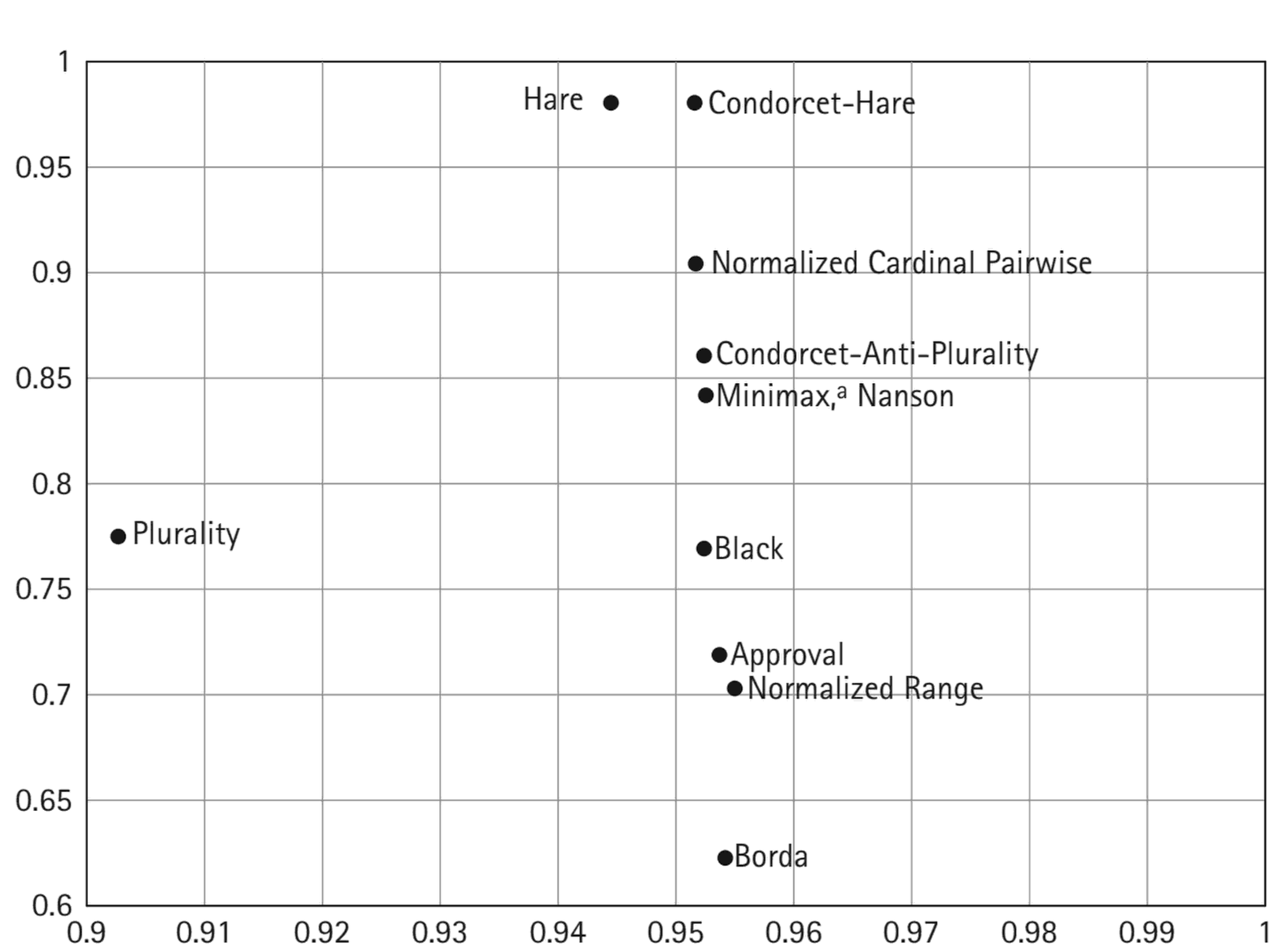

FIGURE 1.2 Sincere Efficiency and Resistance to Strategy in More Detail for 16 Rules

[a] For elections over triples of candidates, Minimax is equivalent to the Simplified Dodgson, Ranked Pairs, Beatpath (Schulze), Kemeny (Kemeny-Condorcet), and Young voting rules.

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The reported sincere efficiency scores are based on treating the candidate with the greatest sum of scores as the one who would be most valuable. It is not reasonable to believe that voters have the ability to make such interpersonally comparable evaluations, however. The weaker assumption that makes the scores worth considering is that the patterns of preferences in the data provide as good an approximation as is possible to obtain of what would be found in interpersonally comparable utilities for candidates—if it were possible to obtain such data.

1.3.1 Plurality

Consider first the most familiar rule: the Plurality rule. This rule allows each voter to give a vote to the candidate of his or her choice, and the candidate with the most votes wins. The principal virtues of Plurality are its familiarity and simplicity. It receives a top score of 5 for ease of voting and a score of 4 for ease of counting, the latter below only that of Random Dictator, which involves considering only one vote. The score of 4.5 for ease of understanding derives from the possibility that people may not understand that a Plurality winner need not win a majority of the votes.

The efficiency score of Plurality, at 0.9027, is about twice as far from full efficiency as most other rules. The resistance to strategy score, at 0.7749, is middling. Plurality is poor in resisting clones because a clone will divide support. With a resistance to tying of 0.9856, it is somewhat better than average at avoiding ties in small electorates. An electorate that is willing to forgo familiarity and simplicity can find many rules that are superior to Plurality in other respects.

1.3.2 Random Dictator and Range

Random Dictator and Range are the rules that achieve perfection in resistance to strategy and sincere efficiency, respectively, but at significant cost. Random Dictator is the rule that a voter is picked at random, and the candidate chosen by that voter is the winner (Zeckhauser 1973). Random Dictator has the virtue of being entirely resistant to strategy. This is because, if a voter's vote counts at all, it determines the outcome completely, so there is no incentive to misstate one's preference. But the cost in sincere efficiency is quite high—a score of just 0.5181. Random Dictator receives top scores for resistance to clones, ease of voting, ease of counting, and ease of understanding. It would generate a tie only if the Random Dictator could not decide and called for a random process.

If there is a voting environment in which it is very important to have sincere votes, while sincere efficiency is of little concern, then Random Dictator might be attractive. When there is concern about sincere efficiency, Random Dictator will not be an attractive rule.

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Range voting requires that each voter give a score to each candidate, and the candidate with the greatest sum of scores wins. Range voting has properties opposite to Random Dictator, with a score of 1 for sincere efficiency but only 0.1814 for resistance to strategy. Range voting receives a score of 5 for resistance to clones and ease of understanding, but only 3 for ease of voting because of the effort involved in assigning scores to all candidates. The score of 3.5 for counting reflects the effort involved in summing the assigned scores. The score of 4 for ease of understanding is less than perfect because of the difficulty of understanding how one might attempt to makes one's own scores comparable to those of others.

The score for Range of 0.9980 for resistance to tying is quite good compared to most other rules. It is in italics in table 1.1 because it is based on an estimate derived by examining the 126,637 triples in the Green-Armytage et al. (2016) data, where there were 28 cases of a tie, a rate of 0.00022. In these "elections," the average number of voters was about 1,000. To achieve an estimate that would be relevant for electorates of 100 voters, as were used in measuring resistance to tying for the other rules, the estimate was multiplied by 10 and then rounded off to one digit, based on the pattern of tie rates for electorates of 100 and 1,000 voters in Plassmann and Tideman (2014). This tie rate is based on the Politbarometer scale of −5 to 5. A scale with more possible scores could be expected to lead to a lower tie rate.

If there is *no* doubt that voters will report their preferences sincerely, and if the voters share an understanding of a scale on which their preferences can be reported as indicators of candidate value, then Range voting is an ideal voting method.

The extreme costs of complete sincere efficiency or complete resistance to strategy make it likely that a rule that compromises, attaining less than full sincere efficiency and less than full resistance to strategy, will be most attractive. We turn next to such compromise rules.

1.3.3 Hare

The Hare rule is named for London barrister Thomas Hare, who suggested, in connection with proportional representation, that the votes of defeated candidates should be reallocated to remaining candidates (Hare 1865; Hoag and Hallet 1926, 176). It now goes by several other names. Its traditional name in the United Kingdom is the Alternative Vote, or AV. In the past 20 years, it has become known in the United States as Instant Run-off Voting (IRV) and as Ranked Choice Voting (RCV). The Hare rule requires that voters rank the candidates, with the ballots assigned initially to the candidates named first on them. If there is a candidate who has a majority of the votes, that candidate is the winner. If not, then the candidate with the fewest votes is eliminated, and the votes for that candidate are reallocated to the candidates named next on those ballots. The process of eliminating the candidate with the fewest votes and reallocating those votes continues until a candidate has a majority of votes.

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The Hare rule is highly resistant to strategy (0.9804). The Gibbard-Satterthwaite theorem (Gibbard 1973; Satterthwaite 1975) shows that every non-dictatorial voting rule for more than two candidates that allows all preferences is susceptible to strategic manipulation in some cases. For the Hare rule, such cases are quite rare. The following formal property of the Hare rule helps explain this rarity.

A candidate, X, is a "Condorcet winner" if, when X is compared, head to head, with every other candidate, there is always a majority that prefers X to the other candidate (Condorcet 1785). If the winning candidate has more than one-third of the first-place votes, then supporters of other candidates have no possible strategy that will cause the winning candidate to have the fewest first-place votes and be eliminated in the first round. If the winning candidate is also a Condorcet winner, then that candidate will necessarily win in the second round with sincere votes, so there will be no second-round majority that will want to defeat that candidate, so that when these conditions are satisfied, there will be no feasible strategy to defeat the candidate.

The rarity of opportunities to manipulate the outcome under the Hare rule is also related to fact that voters' perceptions of the relative positions of candidates tend to not line the candidates up along a single dimension. One generally finds that, comparing three candidates, every candidate is generally last in the rankings of a significant proportion of the voters, and the candidate in the center would be last in none of the rankings if the positions of the candidates lined up along a single dimension. When candidates line up along a single dimension, opportunities to manipulate the result of the Hare rule occur more frequently (Van der Straeten et al. 2010, 451–452).

While Hare is unsurpassed in resistance to strategy (except by Random Dictator), its sincere efficiency score of 0.9445 is lower than those of all the rules discussed here, but only by 0.0105 at most. An important reason for its worse performance in sincere efficiency is that under Hare, a natural compromise candidate can be eliminated for having the smallest number of first-place votes. The fact that Hare is so close behind most other rules in sincere efficiency is a reflection of how seldom this happens in the data.[2]

Hare is given a score of 2.5 for ease of understanding—in the middle of the possibilities. The score of 2.5 for ease of counting is based on the effort involved in counting the votes repeatedly as candidates are eliminated. Like all rules that involve ranking, Hare has a score of 3 for ease of voting—equal to that of Range voting. It is probably harder to rank candidates than to score them. If it were actually necessary to rank candidates, a score of 2.5 for ease of voting would be justified. The score of 3 is based on the idea that voters can score the candidates and the vote-counting process can extract rankings from those scores.

Hare gets full marks in terms of resistance to clones because candidates are eliminated one by one, so that as soon as a clone is eliminated, it is as if the clone were never in the election.

The Hare rule's resistance to tying, at 0.9748, is on the low side.

Under the names Instant Run-off Voting and Ranked Choice Voting, Hare is growing in popularity in the United States as a replacement for Plurality, which it dominates in terms of sincere efficiency and resistance to strategy.

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1.3.4 Condorcet-Hare

The rule that seems best placed to supersede the Hare rule is the Condorcet-Hare rule. This rule was called the Alternative Smith rule in Tideman (2006) because it involves applying the Alternative Vote (the Hare rule) to the Smith set (the top cycle). Green-Armytage (2011) shows that four versions of this rule have been invented independently. The four versions produce the same results for elections among triples of candidates, but can produce different results when there are four or more candidates. The focus here is on the version discussed in Tideman (2006).

Condorcet-Hare is the first rule discussed that is "Condorcet consistent." That is, a voting rule is Condorcet consistent if, as suggested by Condorcet (1785), whenever there is a Condorcet winner, that candidate is chosen. Sometimes there is no Condorcet winner, and different Condorcet-consistent voting rules select different candidates in these cases.

All versions of Condorcet-Hare have the characteristics that if there is a Condorcet winner, then that candidate is chosen, and if there is no Condorcet winner, then some version of the Hare rule is employed. The version of Condorcet-Hare that is discussed in Tideman (2006) specifies that if there is no Condorcet winner, then all candidates not in the "top cycle" are eliminated, and the candidate from the top cycle with the fewest first-place votes is also eliminated. The top cycle (the Smith set) can be identified as the candidate or candidates who beat or tie the largest number of other candidates in head-to-head comparisons, and all who beat or tie them, and all who beat or tie them, and so on, until no more are added. After eliminating the candidates not in the Smith set and one who is in the Smith set, one restarts the vote counting with this smaller number of candidates. The process of counting, eliminating candidates, and restarting the count continues until there is only one candidate left.

The Condorcet-Hare rule has the same score as the Hare rule for resistance to strategy: 0.9804. This equality arises because, as shown by Green-Armytage et al. (2016), the election outcomes that are susceptible to strategic manipulation under the Condorcet-Hare rule are exactly those that are susceptible under the Hare rule, although with different outcomes.

The sincere efficiency of Condorcet-Hare, at 0.9516, is closer than Hare to the other rules shown in figure 1.2, coming within 0.0034 of all of them. The improved performance in terms of sincere efficiency is caused by the fact that, except when dealing with a cycle, Condorcet-Hare does not eliminate a candidate on the basis of the few first-place votes, so that compromise candidates have a better chance of success. This characteristic of Condorcet-Hare might be objectionable, because it might result in a candidate's being elected with little or no first-choice support. This issue is not as serious as one might think, however. Among the 126,637 triples in the Green-Armytage et al. (2016) data, there are 1,029 cases in which (1) there were one or more (possibly tied) Condorcet winners, and (2) the set of winning candidates under the Hare rule was not the same as the set of Condorcet winners. For these cases, an examination of the ratio of the number

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of first-choice votes of the Condorcet winner to the number of votes of the candidate with the most first-choice votes was tabulated. The smallest instance was 0.5398. Thus, it seems that a rule of eliminating any candidate with fewer than half as many first-place votes as the candidate with the most first-place votes would have very little impact, at least in elections with three candidates.

Condorcet-Hare is fully resistant to clones. Its resistance to tying, at 0.9746, is on the low side. Like other rules requiring a ranking of candidates, Condorcet-Hare receives a score of 3 for ease of voting. Its scores for ease of understanding and ease of counting are both 2—half a notch below those of Hare.

The occasions when Hare and Condorcet-Hare are subject to strategy are so rare, and the difficulty of discerning in advance what strategy might be successful is so great, that these rules might in practice be as good as strategy free. The Condorcet-Hare rule has a plausible claim to being the best available rule. This claim is tested by comparing Condorcet-Hare to the remaining rules, beginning with the rules that are most susceptible to strategy.

1.3.5 Borda

The Borda rule (Borda [1781] 1995) specifies that a candidate receive one point for every candidate above whom he or she is ranked by any voter, and the candidate with the most points wins. It was invented in 1770 by Jean-Charles de Borda, a French military officer and scientist. The Borda rule presumes that the diminution in the estimated quality of a candidate associated with a one-step reduction in ranking by a voter is the same for all steps of all voters. If this presumption were universally true, then the Borda rule would have the same sincere efficiency as the Range voting rule. Since it is not universally true, the Borda rule has a lower sincere efficiency than Range voting. However, because the presumption is not terribly far from true on average, only Range and Normalized Range have greater sincere efficiency than the Borda rule's 0.9542.

It is relatively easy to identify the strategy that one would follow to provide an insincere vote that will magnify the impact of one's vote under the Borda rule. If a voter is able to discern which two candidates are most likely to be contenders for winning the election, then the strategy that maximizes the impact of the voter's vote is to place the one of these two that one prefers at the top of one's ballot and the other at the bottom. The frequency with which such a strategy would be successful for supporters of a candidate who would lose with sincere voting is reflected in the relatively low score of the Borda rule (0.6226) in resistance to strategy.

Borda's resistance to tying, at 0.9907, is higher than that of most other rules. This is the result of the fact that by summing the rankings, Borda generates candidate scores that are larger and therefore less likely to be equal. Its resistance to cloning is in the lowest category because the introduction of a clone that is always ranked below a fellow clone improves the relative performance of the fellow clone.

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Borda receives a score of 4 for ease of understanding, which is better than Hare and Condorcet-Hare. Its score for ease of counting is 3.5, the same as for Range. Its score for ease of voting is 3, the same as all rules requiring a ranking or scoring of candidates.

1.3.6 Normalized Range

The Normalized Range rule requires that voters give scores to candidates on a scale on which the most favored candidate receives a score of 100 and the least favored a score of 0, with all other candidates receiving scores of the voter's choice in the range of 0 to 100. The candidate with the most total points wins. Normalized Range is quite similar to Borda. The difference is that Borda assigns equally spaced scores to candidates, while Normalized Range allows voters to space the candidates as they choose. This freedom for voters to space candidates as they choose gives Normalized Range a sincere efficiency of 0.9550—the greatest of all rules other than Range. Normalized Range also has a score for resistance to strategy, at 0.7029, that is noticeably better than that of Borda. This is presumably because when a losing candidate who is a close competitor has received the honest range scores, these scores are generally more than 50, so there is less room for supporters to raise them than under the Borda rule.

The score for Normalized Range in resistance to tying is estimated to be 0.9980, the same as that for Range. Normalized Range gets top marks in independence of clones and, with an ease of understanding score of 3, it is above average. Its ease of counting, at 3.5, is equal to the scores of Range and Borda. Ease of voting is scored at 3, like all rules requiring scores or rankings.

The strategic possibilities that Normalized Range offers are similar to those of Borda. If the candidate that one favors has little apparent chance of being elected, then the expected impact of a vote is maximized by identifying the two top contenders and giving a score of 100 to the contender one favors and a score of 0 to the contender whom one does not favor. For the following reason, a strategic voter is unlikely to find it worthwhile to give any candidate a score other than 0 or 100: Imagine a strategic voter who begins by assigning a score of 100 to his or her most favored candidate and tentative scores of 0 to all other candidates. Then the voter considers giving points to candidates other than the favorite. The reason a voter might do so, even though it would reduce the prospects for the voter's favorite, is that the reduced prospects for the voter's favorite might be more than offset by the expected benefit from improving the prospects of candidates whose points are increased. If such a calculation motivates raising a candidate's score from 0 to 1, then the expected value calculation will be nearly identical for raising the score to 2, then 3, then all the way up to 100. Thus a strategic voter will almost never find it worthwhile to assign a score other than 0 or 100 to a candidate under Range voting. So if one is considering Range voting, then, for the protection of unsophisticated voters, it may be better to have instead a voting rule in which only scores of 0 and 100 are possible. That is the effect of the next rule, Approval voting.

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1.3.7 Approval Voting

Approval voting allows voters to vote for (approve of) as many candidates as they wish, with the candidate with the most votes winning. Approval came to prominence in the twentieth century through the efforts of Stephen Brams and Peter Fishburn in 1978, but it was already used in some elections in Sweden in the eighteen century (Lagerspetz 2016, quoting Mäntylä 1981). Approval is equivalent to a restricted version of Normalized Range, with scores restricted to being either 0 or 100. It can also be regarded as equivalent to a restricted Borda rule, in which ties in rankings are allowed and no candidate can be assigned a position lower than second place. The reduced power of the Approval rule to separate candidates results in a sincere efficiency score of 0.9537, compared to 0.9550 for Normalized Range. At the same time, the reduced power to separate candidates reduces opportunities to vote strategically, with resistance to strategy rising from 0.7029 for Normalized Range to 0.7118 for Approval.

The Approval rule's resistance to tying is estimated to be 0.99, similar to Borda, since it (sometimes) shares Borda's characteristic of adding to candidates' scores for second-place votes. Its resistance to clones and ease of voting are the top score of 5. Ease of understanding is 4.5, like Plurality. Ease of counting, at 4, is below only Random Dictator.

In the evaluation of the Approval rule in terms of resistance to strategy, Green-Armytage et al. (2016) define a sincere vote as one that approved of a candidate if and only if the voter had given the candidate a score of 1 or more, on the scale of −5 to 5. This means that if a voter gave candidates scores of 0, 1, and 5, the sincere vote for that voter would be to approve of the candidates with scores of 1 and 5, and not approve of the candidate with a score of 0. The idea that such a voter might find it much more sensible to approve of only the candidate who had a score of 5 illustrates the difficulty of imagining voting under Approval not being strategic. The need to decide whether to approve intermediate candidates means that it is nearly impossible for a voter under Approval to not vote strategically. Should the voter approve all candidates who are tolerable, or trust others to do that and separate the tolerable candidates into the good and the merely tolerable? There is no right answer. A preference for voting rules that permit an unproblematic identification of what it means to vote sincerely will lead one to favor rules other than Approval voting.

1.3.8 Black

The Black voting rule specifies that if there is a Condorcet winner, then that candidate wins, and if not, then the Borda rule is applied. It was proposed by Duncan Black (1958, 58–59). The principal difficulty with the Black rule is that, while its resistance to strategy, at 0.7692, is a bit better than Borda, Normalized Range, and Approval, it is still rather low. Black's use of the Borda rule when there is no Condorcet winner means that sinking a leading candidate is relatively easy if, by voting strategically, a group can create

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a majority-rule cycle (a situation where A beats B, B beats C, and C beats A by majority rule). Black has a sincere efficiency score of 0.9524, which is lower than that of Approval by 0.0013.

Black's resistance to tying, at 0.9977, is better than all but three rules. This can be attributed to the fact that it has the benefit of summing the rankings, like Borda, but only needs to turn to this when there is a majority-rule cycle. Like Borda, Black does not resist clones. Its complexity gives it a score of 2.5 for ease of understanding, like Hare. Its score for ease of counting, at 3, is between Borda (3.5) and Hare (2.5). Black's ease of voting is 3, like other rules requiring scores or rankings.

1.3.9 Minimax

The Minimax rule specifies that the winner is the candidate whose worst loss in head-to-head comparisons with other candidates is least bad. A Condorcet winner, if one exists, is the only candidate with no losses in head-to-head comparisons, so Minimax elects the Condorcet winner if one exists. Minimax was first described by Duncan Black (1958, 175), in his effort to discern what voting rule Condorcet had in mind, but Black did not advocate Minimax. Minimax is sometimes called the Simpson-Kramer rule because it was advocated by Simpson (1969) and Kramer (1977). The sincere efficiency score of Minimax is 0.9519, which is lower than Black by 0.0005. Its resistance to strategy score is 0.8461, which is better than Black by 0.0769. When resistance to strategy is at the level of Minimax or better, opportunities to benefit from voting strategically are fairly rare, and strategies to benefit from voting strategically are somewhat difficult to identify.

Compared to other voting rules that are at least as good in terms of resistance to strategy and comparable in terms of sincere efficiency, Minimax is probably the simplest. It deserves consideration by an electorate that finds Hare not quite efficient enough and Condorcet-Hare to be too complicated.

Minimax has a resistance to tying on the low side, at 0.9749. It resists pairs but not triples of clones, for a score of 3. Its ease of understanding is 2, like Condorcet Hare. Its ease of counting is 2.5, like Hare. Like other rules requiring rakings or scorings, it has an ease of voting of 3.

The next five rules to be discussed are equivalent to Minimax for elections involving two or three candidates, but they can differ when there are more than three candidates. Because they select the same candidates as Minimax for elections with three candidates, they have the same scores as Minimax for sincere efficiency, resistance to strategy, and resistance to tying. But they are noticeably more complex. Their greatest potential attraction comes from their capacity to identify winners in arguably more rational ways when there are cycles among four or more top candidates. The chances of such cycles go up as the number of candidates increases and as the number of voters decreases.

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1.3.10 Kemeny

The next three rules to be discussed, Kemeny, Ranked Pairs, and Schulze (Beatpath), all focus on identifying a winning ranking and select the candidate at the top of this ranking. Under the Kemeny rule, the winning ranking is the ranking with the smallest sum of majority margins in paired comparisons that deviate from transitive consistency. The winning candidate is the candidate at the top of this ranking. The Kemeny rule was developed in the twentieth century by Kemeny (1959). As Young (1988) explains, Kemeny was probably the rule that the Marquis de Condorcet (1785) had in mind in his essay on voting, thus the rule is sometimes called the Kemeny-Condorcet rule. Like other rules requiring that voters rank the candidates, Kemeny has a score of 3 for ease of voting. The principal difficulty with Kemeny is that if there is a majority-rule cycle involving N candidates at the top of the overall ranking, then it may be necessary to evaluate $N!$ different orderings of the candidates to identify the winning ranking. This can be prohibitively time-consuming if N is very large, giving it a score of 0.5 for ease of counting. This computation difficulty arguably makes Kemeny unacceptable. Kemeny is not resistant to clones; its score on this dimension is 1. Its ease of understanding is 1.5.

1.3.11 Ranked Pairs

The determination of the winner by Ranked Pairs begins by noting the results of all head-to-head contests between pairs of candidates. These results are ordered by the size of the majority and then applied sequentially as possible constraints on the overall ranking of the candidates. The overall ranking is constrained to be consistent with the rankings of pairs in the first two majorities. All rankings of the candidates that are not consistent with the rankings of pairs in the first two majorities are eliminated. Beginning with the third majority, the result is constrained to be consistent with the ranking of the pair, provided that this does not rule out all remaining rankings. The exclusion of rankings continues until just one ranking remains. The candidate at the top of this ranking is the winner.

The Ranked Pairs rule was promoted initially (Tideman 1987) because it is fully independent of clones (a score of 5). Like other rules requiring rankings or scorings, Ranked Pairs has an ease of voting score of 3. Its score for ease of counting is 1.5. Websites such as http://civs.cs.cornell.edu/rp.html include Ranked Pairs among the voting rules that they will employ for users. Its ease of understanding, like Kemeny, is 1.5.

Ranked Pairs might be an attractive voting rule for an electorate that can tolerate complexity, expects to encounter majority-rule cycles involving many candidates, and wants a theoretically satisfying way of dealing with them.

1.3.12 Schulze (Beatpath)

The Schulze voting rule was proposed in Schulze (2003). It operates on the basis of "chains" linking pairs of candidates. A "path" is a sequence of candidates. A path is a

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chain if there is a majority that favors each candidate over the next one. The "strength" of a chain is the size of its smallest majority. If there are majority-rule cycles in a voting outcome, then for every pair of candidates, (*X, Y*), in the cycle, there is both a chain from *X* to *Y* and a chain from *Y* to *X*. For such cases, *X* "disqualifies" *Y* if the strongest chain from *X* to *Y* is stronger than the strongest chain from *Y* to *X*. Schulze (2003) showed that "disqualifies" is a transitive relation. The Schulze rule defines the winner of an election to be the candidate at the top of the "disqualifies" relation.

The Schulze winner is the same as the Ranked Pairs winner in a very large proportion of cases. A great deal of expertise is required to develop a preference between Ranked Pairs and Schulze. The scores for the two rules are the same in all the properties evaluated in this chapter, except that Schulze is lower by half a notch in ease of understanding.

The website http://civs.cs.cornell.edu/rp.html will calculate Schulze, as well as Ranked Pairs winners of elections.

1.3.13 Simplified Dodgson

While discussing voting rules that might be used, Charles Dodgson (quoted in Black 1958, 227–230) suggested that a voting rule is unattractive if it does not elect the candidate who can be turned into a Condorcet winner by the fewest inversions of adjacent candidates on ballots. This has led to the Dodgson rule being defined as the rule that the winner is the candidate who can be turned into a Condorcet winner by the fewest inversions of adjacent candidates on ballots (Fishburn 1973, 172–173). A much less complex rule that generally achieves the same result is the Simplified Dodgson rule (Tideman 2006, 199–201), which evaluates each candidate by the sum of the majorities by which the candidate loses in paired comparisons and declares the winner to be the candidate for whom that sum is smallest. In terms of complexity, Simplified Dodgson is comparable to Minimax. The difference between the two is that if a candidate loses to two candidates, Simplified Dodgson sums the losing margins while Minimax evaluates the candidate by the worst loss. This feature of Simplified Dodgson causes the rule to not resist clones. Simplified Dodgson has the same scores as Minimax in ease of voting, ease of counting, and ease of understanding. Neither Minimax nor Simplified Dodgson accounts for the pattern of losses with the sophistication of Ranked Pairs and Schulze.

1.3.14 Young

The Young voting rule scores each candidate by the number of voters in the largest subset of voters for which the candidate beats all other candidates in head-to-head contests. The candidate with the largest subset wins. This rule emerged from an effort by Peyton Young (1977) to understand what voting rule Condorcet (1785) had in mind. Like Kemeny, Young can become computationally prohibitive, giving it the same score as Kemeny of 0.5 for ease of counting and arguably ruling it out as a feasible voting rule.

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Young is partially resistant to clones, with a score of 3. It has a score of 1 for ease of understanding, like Schulze.

1.3.15 Nanson

The Nanson rule (Nanson 1882) eliminates all candidates with average or below-average Borda scores, and recomputes Borda scores based on the reduced set of candidates. This process continues until only a single candidate remains. Since a Condorcet winner necessarily has an above-average Borda score, Nanson selects the Condorcet winner if there is one. To four decimal places, Nanson, at 0.9519, has the same sincere efficiency as Minimax and its "relatives" (the rules that are equivalent to it for three-candidate elections). In resistance to strategy, at 0.8464, Nanson beats Minimax and its relatives by 0.0003.

In resistance to tying, at 0.9766, Nanson is a bit above Minimax and its relatives, perhaps because of its incorporation of a Borda component. In other characteristics, Nanson is not resistant to clones and has the same scores as Simplified Dodgson.

1.3.16 Condorcet-Anti-Plurality

The Condorcet-Anti-Plurality rule chooses the Condorcet winner, if one exists, and chooses the candidate with the fewest last-place votes, if there is no Condorcet winner. This rule was not devised because anyone thought that it would be a good rule. Rather, it emerged from the efforts of Green-Armytage et al. (2016) to consider possible three-candidate voting rules in certain continuums. It deserves mention because it scores a bit better than Nanson, Minimax, and the Minimax relatives in both sincere efficiency (0.9524) and resistance to strategy (0.8606). Its estimated resistance to tying (0.975) is in the range of Minimax. It is not at all resistant to clones. Like Black, it receives scores of 3 for ease of counting. Like Borda, it receives a score of 4 for ease of understanding.

1.3.17 Normalized Cardinal Pairwise

The Normalized Cardinal Pairwise rule was devised by Green-Armytage (2004) in an effort to find a rule that would do well in both sincere efficiency and resistance to strategy. The rule operates on the same ballot information as Normalized Range. That is, voters score candidates on a scale with 100 for the most favored candidate and 0 for the least favored candidate. But instead of finding the candidate with the highest average score, it focuses on the top cycle. All candidates not in the top cycle are eliminated, so that if there is a Condorcet winner, that candidate wins. If there is no Condorcet winner, then some process (Ranked Pairs or Schulze) is used to select the winning ranking

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and hence the winning candidate from the top cycle. But there is a new wrinkle: the strength of any majority is measured by the sum of the differences in scores between the two candidates that the majority assigned, ignoring the opinions of the minority with respect to that pairing. The reasoning behind this rule is that the results of paired comparisons that are supported by strong opinions should be hard to overturn because, otherwise, overturning them is likely to become the focus of strategic voting.

At 0.9517, the sincere efficiency of Normalized Cardinal Pairwise is lower than that of Minimax and its relatives by just 0.0002, while its resistance to strategy, at 0.9042, is higher by 0.0581. Thus, at some cost in complexity and only the smallest sacrifice of sincere efficiency, Normalized Cardinal Pairwise achieves a substantial improvement in resistance to strategy. But its resistance to strategy is still less than the 0.9804 of Hare and Condorcet-Hare, by 0.0762. Normalized Cardinal Pairwise is estimated to have a resistance to tying of 0.9980, equal to that of Normalized Range. It is fully resistant to clones and has the standard ease of voting score of the ranking-based rules, of 3. Its ease of counting, at 1, is between Kemeny (0.5) and Ranked Pairs (1.5). Its ease of understanding, 0.5, is lower than that for any other rule.

1.4 What Voting Rule Should Be Used?

It is possible to imagine people finding it attractive to use many of the 18 rules that have been discussed in this chapter. A plausible way to proceed is to identify rules that are not attractive and see what is left. In that spirit, one begins by eliminating the rules that are too difficult to understand. If one supposes that these are the rules with scores of 1.5 or less, then this procedure eliminates five rules. Random Dictator can be eliminated for most purposes. Range is more plausible as a rule for aggregating the opinions of judges than voters, so it can be dropped. Unless voters are strongly wedded to tradition and have little concern for sincere efficiency, they will find Plurality unacceptable, so that's best dropped. This reduces the set of possible rules to 10.

Next, one brings in the question of required resistance to strategy. One plausible criterion is that a voting rule should be at least as resistant to strategy as is Plurality. This removes Black, Approval, Normalized Range, and Borda.

Condorcet-Anti-Plurality is a suspicious rule because it arose accidentally. It probably does not deserve to be trusted unless it seems reasonable on the basis of evidence from data involving more than three candidates.

Simplified Dodgson has the potential to be distorted by multiple copies of similar candidates, providing a sufficient reason to exclude it. Nanson's use of Borda for scoring seems unsatisfying, so it goes as well. Now the set of possibilities is down to three rules: Minimax, Hare, and Condorcet-Hare.

Of the three remaining rules, Hare is the simplest. An electorate that wants reasonable sincere efficiency and no unnecessary complexity might choose it. For the greatest sincere efficiency among the remaining rules, the electorate would choose Minimax. For

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resistance to strategy that is as good as Hare, with a sacrifice of only 0.0003 in sincere efficiency compared to Minimax, the electorate would choose Condorcet-Hare if they could tolerate its complexity.

An electorate with different tastes might go through such an exercise and end up with some other rule. In any event, an analysis of this form is recommended. One begins by excluding rules that are clearly unacceptable and then makes the criteria for continued inclusion more and more stringent, until only one rule remains.

The analysis will be more reliable when it can be based on statistics from additional data sources and on actual or simulated elections with more than three candidates.

Notes

* I am indebted to James Green-Armytage and Florenz Plassmann for helpful comments.

1. For reviews of the logical properties of voting rules, see Felsenthal (2012) or Tideman (2006, 165–243).
2. Efficiency is lower when, as in Van der Straeten et al. (2010), the candidates are modeled to lie along a single dimension.

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